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DYNAMICS AND HEAT TRANSFER IN COLLAPSE OF A BUBBLE CONTAINING WET VAPOR IN HYDROPHOBIC LIQUID

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The process of collapse of a bubble containing wet vapor in hydrophobic liquid in the presence of volumetric condensation and external heat transfer is considered. It is shown that external heat transfer can substantially affect the dynamics of the process and damp a shock increase of pressure greatly.

The processes of collapse of vapor bubbles in liquid metal take place in apparatuses of direct energy conversion, in particular, in ejector compressors with a two-phase, two-component working body, with bubble compression occurring in a liquid metal flow. Shock phenomena which bring a threat to the flow part of the structure can occur in bubble collapse; thus, these problems require investigation [1].

It should be noted that the problem of collapse of a cavitation bubble has caught the attention of researchers since Rayleigh, and a number of papers are devoted to it. For example, in [2], dynamic and heat- and mass-transfer processes in collision and growth of vapor bubbles in the proper liquid were considered; in this case, the vapor in the bubble was dry and its condensation took place on the phase interface of the bubble. Unfortunately, this scheme of vapor condensation does not describe the processes of vapor bubble collapse in hydrophobic liquid — low-temperature liquid metal, e.g., indium–bismuth eutectics, serves as an example of this liquid. By virtue of this, in what follows we consider another model of phase transition, where condensation of vapor, e.g., freon, takes place inside a bubble on microdroplets of its own liquid.

Thermophysical processes accompanying the collapse of a vapor bubble can dominate over hydrodynamic processes. In fact, the phase interface, which moves at a velocity w_0, during the process period $\Delta\tau$ shifts at a distance $w_0\Delta\tau$ and changes the bubble volume by $\Delta V = 4\pi a^2 w_0 \Delta\tau$. In isobaric-isothermal condensation, the change in the vapor mass is $\Delta m = 4\pi a^2 w_0 \Delta\tau \rho_g$ and the heat of phase transition $Q_g = l\Delta m$ is released. During the same period of time, the heat $Q_\Sigma = (l_{liq}/a)(T_g - T_{liq})4\pi a^2 \Delta\tau$ is withdrawn by convection from the phase interface to the surrounding liquid.

It is evident that, when $Q_g << Q_\Sigma$, external heat transfer does not influence the processes in the bubble itself and the bubble collapses in the dynamic mode.

When $Q_g >> Q_\Sigma$, external heat transfer does not manage to withdraw the heat of phase transition and the bubble becomes adiabatic. The collapse of the bubble is impossible under these conditions — damping oscillations originate, on cessation of which the condensation of vapor occurs in a purely thermal mode.

Finally, when $Q_g \approx Q_\Sigma$, the bubble collapse is determined by the inertia of dynamic and thermophysical processes.

The ratio of these heat fluxes leads to the following parameter of the problem:

$$\frac{Q_g}{Q_\Sigma} = \frac{aw_0}{\dfrac{\lambda_{liq}}{\rho_{liq} c_{liq}}} \frac{1}{\dfrac{c_{liq}(T_g - T_{liq})}{l} \dfrac{\rho_{liq}}{\rho_g}} = \frac{\text{Pe}}{\text{Ja}} .$$

When Pe/Ja << 1, the bubble collapses in the inertial mode, when Pe/Ja >> 1 — in the thermal mode, and when Pe/Ja ~ 1, the bubble collapse is formed by both dynamic and thermophysical processes.

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We derive the equation of heat influx for a bubble with wet vapor. From the first law of thermodynamics for a simple thermodynamic system written for 1 m^3 of substance, $\rho_g \delta q = \rho_g di_g - dp$; after differentiation with respect to time τ and with account for the equality $\frac{d}{d\tau}(\rho_g \delta q) = -\text{div}\,\mathbf{q}$ we obtain

$$\rho_g \frac{di_g}{d\tau} = -\,\text{div}\,\mathbf{q} + \frac{dp}{d\tau} . \tag{1}$$

From the continuity equation for wet vapor we find the derivative $d\rho_g/d\tau = -\rho_g\,\text{div}\,\mathbf{w}_g$, with account for which we transform Eq. (1) as

$$\frac{d}{d\tau}(\rho_g i_g) + \rho_g i_g\,\text{div}\,\mathbf{w}_g = -\,\text{div}\,\mathbf{q} + \frac{dp}{d\tau} . \tag{2}$$

In collapse, the vector fields in the bubble and the surrounding liquid are spherically symmetric $\mathbf{w}_i\{w_i, 0, 0\}$, $\mathbf{q}_i\{q, 0, 0\}$; therefore, in the spherical system of coordinates we write for the divergence of the vectors of velocity and density of the heat flux

$$\text{div}\,\mathbf{w}_g = \frac{1}{r^2}\frac{\partial}{\partial r}(r^2 w_g) , \quad \text{div}\,\mathbf{q}_g = \frac{1}{r^2}\frac{\partial}{\partial r}(r^2 q_g) .$$

We make the assumption that in vapor condensation the function $\rho_g i_g$ remains uniform inside a bubble; then Eq. (2) takes the form

$$r^2 \frac{d}{d\tau}(\rho_g i_g) + \frac{\partial}{\partial r}(\rho_g i_g r^2 w_g) = \frac{\partial}{\partial r}\left(r^2 \lambda_g \frac{\partial T_g}{\partial r}\right) + r^2 \frac{dp}{d\tau} . \tag{3}$$

We integrate (3) with respect to the radius going from 0 to a (the bubble radius), allow for the conditions at the center and on the surface of the bubble

$$w_g(0) = 0 , \quad \left.\frac{\partial T_g}{\partial r}\right|_{r=0} = 0 , \quad w_g(a) = w_a , \quad -\lambda_g \left.\frac{\partial T_g}{\partial r}\right|_{r=a} = -\lambda_{\text{liq}} \left.\frac{\partial T_{\text{liq}}}{\partial r}\right|_{r=a} , \tag{4}$$

and obtain the equation of heat influx to the bubble:

$$\frac{d}{d\tau}(\rho_g i_g) + 3\frac{\rho_g i_g w_a}{a} = 3\frac{\lambda_g}{a}\left.\frac{\partial T_g}{\partial r}\right|_{r=a} + \frac{dp}{d\tau} . \tag{5}$$

At some stages of compression the vapor in the bubble can go over to the superheated state. If the vapor pressure is much smaller than critical, the vapor still obeys the Mendeleev–Clapeyron equation. In this case, $\rho_g i_g = \rho_g c_p T_g = \frac{\gamma}{\gamma - 1} p$ and Eq. (5) changes over to the equation for gas pressure in the bubble [2]:

$$\frac{dp}{d\tau} = 3(\gamma - 1)\frac{\lambda_{\text{liq}}}{a}\left.\frac{\partial T_{\text{liq}}}{\partial r}\right|_{r=a} - 3\gamma\frac{p w_a}{a} . \tag{6}$$

We apply the equation of heat influx (5) to wet vapor. The density of wet vapor is $\rho_g = 1/v_g$ and its specific volume v_g and enthalpy i_g are determined by the equalities (the effect of surface curvature is neglected)

$$v_g = (1 - \chi)\,v' + \chi v'' , \quad i_g = (1 - \chi)\,i' + \chi i'' ,$$

where $'$ and $''$ refer to the parameters of saturated liquid and dry saturated vapor on the boundary curve and $\chi = (v_g - v')/(v'' - v')$.

We calculate the derivative from Eq. (5):

$$\frac{d}{d\tau}(\rho_g i_g) = \frac{1}{v_g}\frac{di_g}{d\tau} - \frac{i_g}{v_g^2}\frac{dv_g}{d\tau}. \tag{7}$$

The state of wet vapor is specified by two parameters: the degree of dryness χ and the saturation temperature T, i.e., $v_g(\chi, T)$ and $i_g(\chi, T)$; the total differentials of these functions are

$$dv_g = \left(\frac{\partial v_g}{\partial \chi}\right)_T d\chi + \left(\frac{\partial v_g}{\partial T}\right)_\chi dT, \quad di_g = \left(\frac{\partial i_g}{\partial \chi}\right)_T d\chi + \left(\frac{\partial i_g}{\partial T}\right)_\chi dT, \tag{8}$$

and the partial derivatives are found from the form of the functions v_g and i_g:

$$\left(\frac{\partial i_g}{\partial \chi}\right)_T = i_g''(T) - i_g'(T) = l(T), \quad \left(\frac{\partial i_g}{\partial T}\right)_\chi = \frac{di'}{dT} + \chi\frac{dl}{dT} = c' + \chi\frac{dl}{dT},$$

$$\left(\frac{\partial v_g}{\partial \chi}\right)_T = v''(T) - v'(T), \quad \left(\frac{\partial v_g}{\partial T}\right)_\chi = \frac{dv'}{dT} + \chi\frac{d}{dT}(v'' - v'),$$

with all thermodynamic functions on the boundary curve being dependent solely on the temperature of saturation.

We substitute these derivatives into the expressions for the total differentials (8); then we calculate the derivative (7) and substitute it into Eq. (5); as a result we have

$$\left\{\left[c' + \chi\frac{dl}{dT} - \frac{i_g(\chi, T)}{v_g(\chi, T)}\left(\frac{dv'}{dT} + \chi\frac{d}{dT}(v'' - v')\right)\right]\frac{T(v'' - v')}{l} - v_g(\chi, T)\right\}\frac{dp}{d\tau} =$$

$$= 3\frac{v(\chi, T)\lambda_{liq}}{a}\left.\frac{\partial T_{liq}}{\partial r}\right|_{r=a} - 3\frac{i_g(\chi, T)w_a}{a} - \left(l - \frac{i_g(\chi, T)}{v_g(\chi, T)}(v'' - v')\right)\frac{d\chi}{d\tau}. \tag{9}$$

Here, in transformations we used the Clapeyron–Clausius equation

$$\frac{dp}{dT} = \frac{l}{T(v'' - v')}. \tag{10}$$

The equation of heat influx (9) involves the following unknowns: p, χ, T, w_a, a, and $\lambda_{liq}(\partial T_{liq}/\partial r)\big|_{r=a}$, i.e., six in all. Consequently, this equation must further be supplemented by five equations. One of these is the above-mentioned Clapeyron–Clausius equation (10), and its solution is (it is taken that within a narrow range of pressures l = const)

$$\frac{p(T)}{p_0} = \exp\left[\frac{l}{R_\mu}\left(\frac{1}{T_0} - \frac{1}{T}\right)\right], \tag{11}$$

where the parameters with the subscript 0 refer to some reference point on the phase curve of saturation. Within a wide range of pressures, up to the critical one, Eq. (10) can be integrated numerically as well, if the function $l(T)$ is known.

We find the relation between the degree of vapor dryness χ and the bubble parameters. Wet vapor is a mixture of liquid droplets and dry saturated vapor. At saturation pressures far from critical, the dry saturated vapor still obeys the Mendeleev–Clapeyron equation. Let the bubble with an initial radius r_0 be filled with the dry saturated vapor, i.e., $\chi = 1$. The vapor pressure in the bubble is

$$p = p_{s0} - 2\,\frac{\Sigma}{a}\,\frac{v_{liq}}{v_g - v_{liq}} \approx p_{s0}\,.$$

Since, in the model adopted, the condensation of vapor occurs inside the bubble but not on its surface, during the collapse the bubble mass remains constant at any degree of dryness:

$$m = \frac{4}{3}\,\pi\xi_0^3\rho_{liq} = \frac{4}{3}\,\pi a_0^3\,\frac{p_{s0}}{R_\mu T_{s0}}\,, \tag{12}$$

where ξ_0 is the bubble radius in the case of complete condensation of vapor in it, i.e., at $\chi = 0$.

During the collapse, the gas component of the wet vapor can be within the volume $V_v - V_{liq}$. Then, the Mendeleev–Clapeyron equation is written in the form $p(V_v - V_{liq}) = \chi m R_\mu T$ or, with allowance for the relations $V_{liq} = (1 - \chi)m/\rho_{liq}$ and $V_v = (4/3)\pi a^3$, as

$$pa^3 - (1 - \chi)\,\xi_0^3 p = \chi p_{s0} a_0^3/T_{s0}\,.$$

If we introduce the given variables $a_* = a(\tau)/a_0$, $p_* = p(\tau)/p_{s0}$, and $\vartheta_* = T(\tau)/T_{s0}$, then the above equation is rewritten as

$$p_* a_*^3 - (1 - \chi)\,p_*\,(\xi_0/a_0)^3 = \chi\vartheta_*\,,$$

whence we find the degree of dryness

$$\chi = \frac{p_*\,(a_*^3 - (\xi_0/a_0)^3)}{\vartheta_* - p_*\,(\xi_0/a_0)^3}\,. \tag{13}$$

In particular, if the condensation of vapor in the bubble follows the isobar, $p_* = 1$, and $\vartheta_* = 1$, then for χ from (13) we obtain the expression $\chi = (a_*^3 - (\xi_0/a_0)^3)/(1 - (\xi_0/a_0)^3)$. On the right-hand branch of the boundary curve liquid–vapor $a_* = 1$ and $\chi = 1$ and on its left-hand branch $a_* = (\xi_0/a_0)$ and $\chi = 0$.

In the case where the bubble begins to collapse from the state of saturated vapor, and consequently, its initial radius a_0 does not correspond to the vapor state on the right-hand branch of the boundary curve, it must be recalculated by the condition of mass constancy (12):

$$\left(\frac{a_{s0}}{a_0}\right)^3 = \frac{p_0/p_{s0}}{T_0/T_{s0}} = \frac{p_{*0}}{T_{*0}}\,.$$

The following relation, which supplements (9), is the Rayleigh–Lamb equation [2] (it characterizes the dynamics of the bubble):

$$a\,\frac{d^2a}{d\tau^2} + \frac{3}{2}\left(\frac{da}{d\tau}\right)^2 + 4\nu_{liq}\,\frac{1}{a}\,\frac{da}{d\tau} + 2\,\frac{\Sigma}{\rho_{liq}a} = \frac{p\,(\tau) - p_\infty}{\rho_{liq}}\,, \tag{14}$$

where $p(\tau)$ is the vapor pressure in the bubble and p_∞ is the pressure at infinity in the surrounding liquid.

In addition to the equations presented, it is necessary, in order to describe the process, to invoke the equation of heat influx for the surrounding liquid:

$$\rho_{\text{liq}} c_{\text{liq}} \left(\frac{\partial T_{\text{liq}}}{\partial \tau} + w_{\text{liq}} \frac{\partial T_{\text{liq}}}{\partial r} \right) = \frac{1}{r^2} \frac{\partial}{\partial r} \left(r^2 \lambda_{\text{liq}} \frac{\partial T_{\text{liq}}}{\partial r} \right) + 12 \mu_{\text{liq}} \frac{w_{\text{liq}}^2}{r^2} , \tag{15}$$

where the radial velocity $w_{\text{liq}}(r, \tau)$ is related to the velocity of the phase interface in terms of the continuity equation:

$$w_{\text{liq}} (r, \tau) = w_a (\tau) \left(\frac{a}{r} \right)^2 . \tag{16}$$

Expressions (9), (10), (13), (14), and (15) with $w_a = a$, and relation (16) represent the closed system of equations which describes thermal and hydrodynamic processes in the collapse of the cavitation bubble.

Let the vapor bubble be first in the equilibrium state and its parameters correspond to dry saturated vapor:

$$\tau = 0 : \quad p (0) = p_{\text{s0}} , \quad T (0) = T_{\text{s0}} , \quad a (0) = a_{\text{s0}} , \quad p_{\text{s0}} - p_\infty = 2 \frac{\Sigma}{a_{\text{s0}}} .$$

Then at infinity there arises the pressure jump δp_∞, which disturbs the system from the equilibrium. We represent the right-hand side of (14) as

$$p (\tau) - p_\infty = p (\tau) - p (0) + p (0) - p_\infty - \delta p_\infty = p (\tau) - p (0) + 2 \frac{\Sigma}{a_{\text{s0}}} - kp (0) = p (0) (p_* - (k + 1)) + 2 \frac{\Sigma}{a_{\text{s0}}} ;$$

when $k > 0$ the bubble collapses, and when $-1 < k < 0$ it grows. In the general case, $k = k(\tau)$.

With account for the introduced disturbance parameter it is convenient to present Eq. (14) in the dimensionless form

$$a_* \frac{d^2 a_*}{d\tau_*^2} + \frac{3}{2} \left(\frac{da_*}{d\tau_*} \right)^2 + \frac{4}{\text{Re}} \frac{1}{a_*} \frac{da_*}{d\tau_*} + \frac{2}{\text{We}} \frac{1 - a_*}{a_*} = p_* - (k + 1) , \tag{17}$$

where

$$\text{Re} = \frac{a_{\text{s0}} w_0}{\nu_{\text{liq}}} , \quad \text{We} = \frac{\rho_{\text{liq}} w_0^2}{\Sigma / a_{\text{s0}}} ,$$

Here $\tau_* = \tau / \tau_0$ is the reduced time and $w_0 = \sqrt{p(0) / \rho_{\text{liq}}}$ and $\tau_0 = a_{\text{s0}} / w_0$ are the characteristic values of the velocity and time.

We pass over to the mobile system of coordinates $\eta_* = r/a(\tau)$ and by the transition formulas [2]

$$\frac{\partial}{\partial r} ( ) = \frac{1}{a (\tau)} \frac{\partial}{\partial \eta_*} ( ) , \quad \left(\frac{\partial}{\partial \tau} \right)_r = \left(\frac{\partial}{\partial \tau} \right)_{\eta_*} - \frac{\dot{a}}{a} \eta_* \frac{\partial}{\partial \eta_*} ( ) ,$$

and expression (16) we transform (15) as

$$\frac{\partial \theta_*}{\partial \tau_*} + \frac{\dot{a}_*}{a_*} \left(\frac{1}{\eta_*^2} - \eta_* \right) \frac{\partial \theta_*}{\partial \eta_*} = \frac{1}{\text{Pe}} \frac{1}{a_*^2} \left(\frac{\partial^2 \theta_*}{\partial \eta_*^2} + \frac{2}{\eta_*} \frac{\partial \theta_*}{\partial \eta_*} \right) + 12 \frac{\text{Ec}}{\text{Re}} \left(\frac{\dot{a}_*}{a_*} \right)^2 \frac{1}{\eta_*^6} , \tag{18}$$

where $\theta_* = T_{\text{liq}}(r, \tau) / T_{\text{liq0}}$ is the reduced temperature in the liquid phase and

$$\text{Pe} = \frac{a_{\text{s0}} w_0}{\dfrac{\lambda_{\text{liq}}}{\rho_{\text{liq}} c_{\text{liq}}}} , \quad \text{Ec} = \frac{w_0^2}{c_{\text{liq}} T_{\text{liq0}}} .$$

We express the derivative $dp/d\tau$ from (9) and write it a reduced form:

$$\frac{dp_*}{d\tau_*} = \frac{1}{p_{s0} F(\chi, T)} \left[3 \frac{\lambda_{liq} T_{s0} \tau_0}{a_{s0}^2} v_g(\chi, T) \frac{1}{a_*^2} \left. \frac{\partial \theta_*}{\partial \eta_*} \right|_{\eta_*=1} - 3 i_g(\chi, T) \frac{w_*}{a_*} - \left(l - i_g(\chi, T) \frac{v'' - v'}{v_g(\chi, T)} \right) \frac{d\chi}{d\tau_*} \right]. \quad (19)$$

Here, the function $F(\chi, T)$ is determined by the equality

$$F(\chi, T) = c'(T) + \chi \frac{dl}{dT} - \frac{i_g(\chi, T)}{v_g(\chi, T)} \left(\frac{dv'}{dT} + \chi \frac{d}{dT}(v'' - v') \right) \frac{T(v'' - v')}{l} - v_g(\chi, T) .$$

In the case of saturated vapor, Eq. (6), which in the reduced variables has the form

$$\frac{dp_*}{d\tau_*} = 3(\gamma - 1) \frac{\lambda_{liq} T_{liq} \tau_0}{p_{s0} a_0^2} \frac{1}{a_*^2} \left. \frac{\partial \theta_*}{\partial \eta_*} \right|_{\eta_*=1} - 3\gamma \frac{p_* w_*}{a_*} ,$$

must be used instead of (19).

We also present the Clapeyron–Clausius equation in the dimensionless form

$$\frac{dp_*}{d\vartheta_*} = \frac{l}{p_{s0}(v'' - v')} , \quad (20)$$

where $\vartheta_* = T/T_{s0}$ is the reduced temperature of the vapor in the bubble.

We formulate the initial conditions for the unknown functions:

$$\tau_* = 0: \quad a_* = 1 , \quad w_* = 0 , \quad \theta_*(0, \eta_*) = 1 , \quad \vartheta_*(0) = 1 , \quad p_* = 1 , \quad \eta_* \geq 1 . \quad (21)$$

The conditions at infinity are

$$\eta_* = \infty : \quad \theta_*(\tau_*, \infty) = 1,$$

the conditions on the bubble surface are

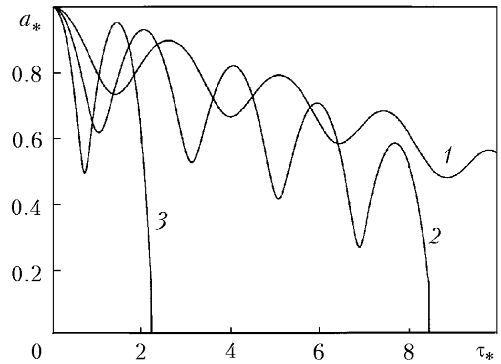
$$\eta_* = 1 : \quad \theta(\tau_*, 1) = \vartheta_*(\tau_*) , \quad q_* = - \left. \frac{\partial \theta_*}{\partial \eta_*} \right|_{\eta_*=1} , \quad w_* = \dot{a}_* .$$

Moreover, when $k > 0$, $w_* < 0$, and $0 \leq \chi \leq 1$ the bubble compresses and when $k < 0$, $w_* > 0$, and $\chi < 1$ it grows.

The derived system of equations can be integrated numerically. The ordinary differential equations of the system were integrated by the Runge–Kutta scheme of the fourth order of accuracy, and Eq. (18), which provides a relation with other equations of the system through the heat-flux density on the bubble surface, was solved by the finite-difference method according to the explicit scheme

$$\theta_{n+1,m} = \theta_{n,m} + \frac{\delta\tau}{(\delta\eta)^2} \left[-\frac{w_n}{a_n} \left(\frac{1}{(m\delta\eta)^2} - m\delta\eta \right) (\theta_{n,m+1} - \theta_{n,m}) \, \delta\eta + \frac{1}{\mathrm{Pe} \, a_n^2} \left(\theta_{n,m+1} - 2\theta_{n,m} + \theta_{n,m-1} + \right. \right.$$

$$\left. \left. + \frac{2}{m} (\theta_{n,m+1} - \theta_{n,m}) \, \delta\eta \right) + 12 \frac{\mathrm{Ec}}{\mathrm{Re}} \left(\frac{w_n}{a_n} \right)^2 \frac{1}{m^6 \delta\eta^4} \right],$$



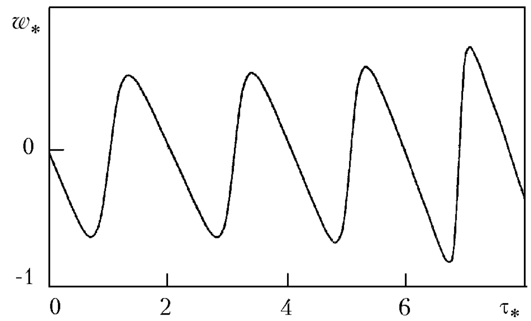


Fig. 1. Dependence of the reduced radius a_* of the vapor bubble on the reduced time τ_* at different disturbance actions: 1) $k = 0.5$; 2) 1; 3) 2.

Fig. 2. Dependence of the reduced velocity of the phase interface w_* on the reduced time τ_* at $k = 1$.

where $\delta\tau = \tau_*/N$, $\delta\eta = L_*/M$, L_* is an infinitely remote point (in the calculations $L_* = 2$), and N and M we selected experimentally, proceeding from the stability of the computation process. Most calculations in the collapse mode were made at $N = 10{,}000$ and $M = 10$ and in the evaporation mode at $N = 50{,}000$ and $M = 20$. The grid functions are $a_n = a_*(n\delta\tau)$, $w_n = w_*(n\delta\tau)$, $\theta_{n,0} = \vartheta_*$, $n \in [0, N]$, and $m \in [0, M]$. Below, we discuss the results of the numerical study of the reduced system of equations.

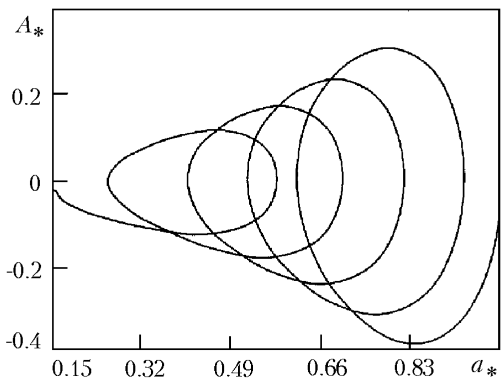
Calculations were made for Freon F-11 bubbles with properties according to [3] in the medium of liquid gallium: the initial radius of the vapor bubble was set equal to $a_0 = a_{s0} = 3{\cdot}10^{-3}$ m; thus the droplet radius was $\xi_0 = 7.81{\cdot}10^{-5}$ m and $c_{\text{liq}} = 4.18$ kJ/(kg·K). The scales of the quantities were $w_0 = 1.53$ m/sec and $\tau_0 = 1.96{\cdot}10^{-3}$ sec, the problem criteria Re = 4590, Pe = 31,800, and Ec = $2.75{\cdot}10^{-5}$, and the disturbance parameter k was varied in the calculations.

Figure 1 presents the radius–time dependence in the action of different pressure pulses on the system. The intersection of the curves with the abscissa axis corresponds to the time of complete condensation of the vapor in the bubble with a finite reduced radius $\xi_0/a_0 = 0.026$. The total reduced time of bubble collapse is $\tau_* > 10$ (curve 1), $\tau_* = 8.4$ (2), and $\tau_* = 2.2$ (3). The radius–time curve indicates that in the process of vapor bubble collapse there take place volume oscillations, with the amplitude of oscillations being the larger the higher the disturbing effect.

Figure 2 depicts, in the reduced coordinates, the velocity–time graph for the vapor bubble collapsing due to the pressure disturbance at $k = 1$. As in the previous case, the velocity of the phase interface also experiences oscillations with increasing amplitude. This special feature revealed on vapor bubbles is not typical of damping oscillations of gas bubbles. The fact that the duration of the phase of bubble compression is larger than that of the expansion phase engages our attention; this indicates greater sluggishness of the process of vapor condensation (compression) as compared with the processes of evaporation (expansion). At the compression stage there takes place vapor superheating, accompanied by pressure increase, which impedes the collapse of the bubble. The reduced velocity of the phase interface reaches the value $w_* \sim 1$ (or $w \sim 1.5$ m/sec); this value is two orders smaller than those in the case of collapse obtained by the scheme of the inertia mode.

Figure 3 shows the time–distance curve of the volumetric flow rate $A_* = w_* a_*^2$ of the phase interface on the bubble radius at $k = 1$. It is seen from the figure that in each oscillation the bubble passes through the point of maximum flow rate, which decreases with the number of oscillations. In particular, the first maximum is attained at $a_{*\text{cr}} = 0.83$ and is $A_{*\text{cr}} = 0.395$. The pressure in the bubble during the collapse is characterized by Fig. 4. Overshoots in the pressure correspond to the moments of highest bubble compression and multiply exceed the initial pressure. A purely inertia mode of bubble collapse with Pe/Ja << 1 precedes the phase of increased pressure in the bubble and that with Pe/Ja >> 1 the phase of maximum pressure, which indicates a thermal mode. In particular, the last oscillation of the bubble is accompanied by the overshoot in pressure to $p_{*\max} = 30$ (curve 3).

Following the pressure, the vapor temperature also experiences oscillations; the character of the variation of the vapor temperature fully corresponds to the pressure curve.



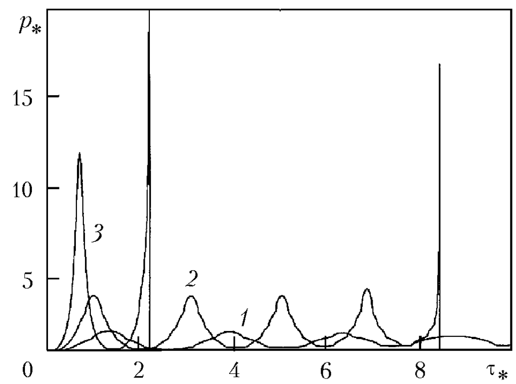


Fig. 3. Dependence of the reduced flow rate of the phase interface A_* of the bubble on the radius a_* at $k = 1$.

Fig. 4. Dependence of the reduced vapor pressure p_* in the bubble on the reduced time τ_*: 1) $k = 0.5$; 2) 1; 3) 2.

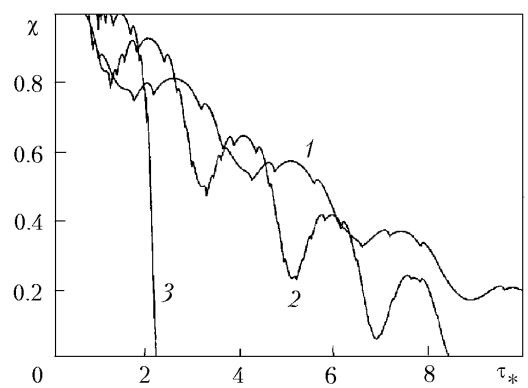


Fig. 5. Dependence of the degree of dryness of the vapor in the bubble χ on the reduced time τ_*: 1) $k = 0.5$; 2) 1; 3) 2.

The amplitude of vapor temperature oscillations increases with increase in the disturbing pressure, with their maxima falling at the moments of highest compression of the bubble. The maximum value of the reduced temperature at $k = 1$ reaches $\vartheta_* = 1.5$, which corresponds to $176^{\circ}C$ at an initial vapor temperature of $23^{\circ}C$. At the moments when the highest temperature is attained in the bubble, intense withdrawal of heat to the surrounding liquid begins.

Figure 5 presents the most important parameter — the dependence of the degree of vapor dryness on time. The curves reveal the hidden special feature in the process of vapor bubble collapse: the evaporation of the condensate occurs in the bubble during some phases. As a whole, the degree of dryness decreases as the bubble collapses, thus reaching a zero value in the complete condensation of vapor. It is noteworthy that the latter state of the vapor in the bubble is not equilibrium but resembles the state of a shock-compressed liquid droplet.

The information on heat exchange between the bubble and the surrounding liquid is presented predominantly by the Nusselt number. Actually, on the one hand, the heat-flux density on the bubble surface is

$$q_{\Sigma} = -\lambda_{\text{liq}} \left(\frac{\partial T_{\text{liq}}}{\partial r}\right)_{r=a} = -\frac{\lambda_{\text{liq}} T_{\text{liq}}}{a_{s0} a_*} \frac{\partial \theta_*}{\partial \eta_*}\bigg|_{\eta_*=1} ,$$

and, on the other hand, it is expressed by the Newton–Richman law:

$$q_{\Sigma} = \alpha \left(T_{\text{liq}0} - T_{\text{g}}\right) = \alpha T_{\text{liq}0} \left(1 - \vartheta_*\right) .$$

Equating these expressions, for the Nusselt number on the bubble surface we obtain (Fig. 6)

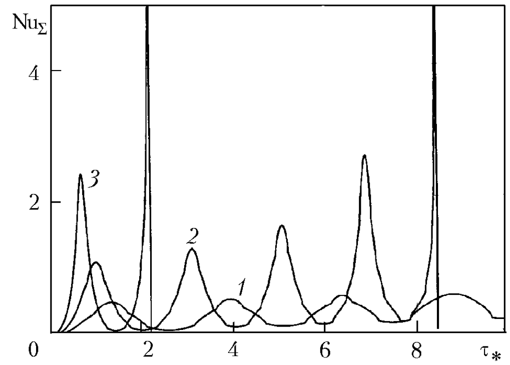


Fig. 6. Dependence of the Nusselt number Nu_Σ on the reduced time τ_*: 1) k = 0.5; 2) 1; 3) 2.

$$Nu_\Sigma = \frac{\alpha a_{s0}}{\lambda_{liq}} = -\frac{1}{a_* \,(1-\vartheta_*)} \left. \frac{\partial \theta_*}{\partial \eta_*} \right|_{\eta_*=1} .$$

The Nusselt number follows the oscillations of the volume of the bubble during its collapse. The increase in the amplitude of the Nusselt number depending on the number of oscillations attracts our attention. The maximum value corresponds to the time instants of highest compression of the bubble and the minimum value to the highest expansion. The increase in the amplitude of Nu_Σ can, apparently, be explained by the decrease in the bubble radius in compression and by the maximum temperature gradient at these time instants. Positive values of Nu_Σ point to the fact that the heat flux is unidirectional — from the bubble to the liquid.

By and large, the temperature field in the surrounding liquid remains uniform. However, the liquid layers adjacent to the phase interphase at a distance of about one to two space steps (1/10 of the radius) experience fluctuations of the temperature in accordance with the vapor temperature: the highest heating of these layers is reached at the instants of highest compression of the bubble.

The results obtained allow estimation of the shock increase of the pressure in collapse of the cavitation bubble. The hydraulic shock from the vapor bubble arises after passing the point of maximum flow rate (see Fig. 3).

The kinetic energy of the surrounding liquid at these instants is

$$E = \int_{V_v}^{\infty} \frac{1}{2} \rho_{liq} w_{max}^2 dV = 2\pi\rho_{liq} \frac{A_{*cr}^2}{a_{*cr}} a_0^3 w_0^2 , \quad A_{*cr} = a_{*cr}^2 w_* ;$$

the potential energy of the shock-compressed liquid in the volume V_v = $3/4\pi a_{*cr}^3$ is determined from the Hooke law

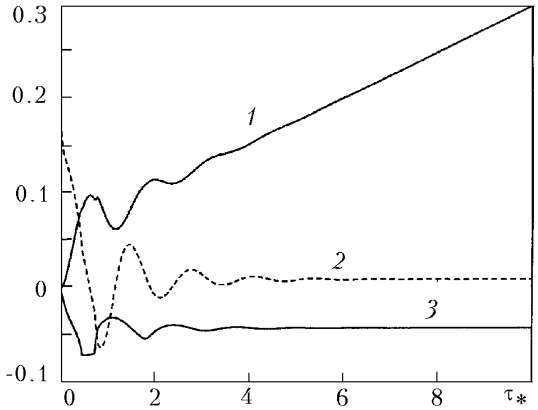
$$\Pi = \frac{1}{2} \chi_{el} p_{max}^2 \frac{4}{3} \pi a_{*cr}^3 a_0^3 .$$

Equating the energies, for the shock pressure we find

$$p_{max} = \frac{A_{*cr}}{a_{*cr}^2} \sqrt{3\rho_{liq} E_{el}}\; w_0 .$$

The calculation for liquid gallium by this formula by the first critical point of the flow rate–radius curve A_{*cr} = 0.395, a_{*cr} = 0.83, w_0 = 4.05 m/sec, and E_{el} = $2.25 \cdot 10^9$ 1/Pa (Fig. 3) gives p_{max} = $1.41 \cdot 10^7$ Pa. (If we relate the elastic potential energy to the volume of the liquid droplet obtained after complete condensation of the vapor, we find for the shock pressure p_{max} = $8.6 \cdot 10^{12}$ Pa.)

The above-given system of equations of dynamics and heat transfer of the bubble with wet vapor describes the process of boiling-up of a saturated liquid droplet in pressure release. As an example, we calculated the system a



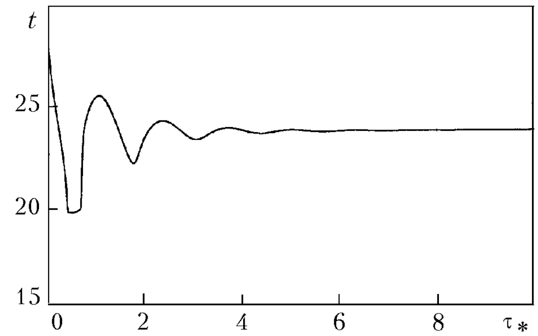


Fig. 7. Dependence of the degree of dryness of the vapor in the bubble $\chi \cdot 10^3$ (1), the number (Pe/Ja)/20 (2), and the Nusselt number (3) on the reduced time τ_* of the process in bubble growth and at $k = -0.2$.

Fig. 8. Dependence of the temperature of the vapor in the bubble t on the reduced time τ_* in pressure release and at $k = -0.2$.

droplet of liquid Freon F-11–liquid gallium, which initially is in equilibrium at a temperature $t_0 = 30^oC$ ($p_{s0} = 1.26 \cdot 10^5$ Pa). Then, in the surrounding liquid, the pressure decreased jumpwise by $\delta p = 0.256 \cdot 10^5$ Pa ($k = -0.2$) and the droplet began to boil-up, thus forming a bubble with wet vapor. The initial radius of the droplet was set to be equal to $\xi_0 = 1$ mm.

Figure 7 illustrates some parameters which characterize the process of bubble growth. The curve of the degree of vapor dryness shows that in boiling-up of the liquid droplet the oscillations of the parameter arise only at the beginning of the process, when Pe/Ja ≈ 1; then, as this number decreases, the oscillations damp and the bubble continues to grow in a purely thermal mode. The Nusselt number (curve 3) follows the oscillations of the bubble parameters (radius, pressure, temperature) at the dynamic stage of the process; at the thermal stage of the process the Nusselt number takes the constant value $Nu_\Sigma = -0.0426$. Within the entire time ranges the heat flux is directed from the liquid to the bubble.

Figure 8 shows the dependence of the temperature of the vapor in the bubble on time in pressure release. At the thermal stage, a constant temperature head of about 6^oC, which maintains the bubble growth, reaches a steady state.

The model considered also describes the collapse of the cavitation bubble in its own liquid, if we assume that all vapor which condensed on the phase interphase is related to the mass of the bubble but not to the surrounding liquid.

CONCLUSIONS

1. Relaxation of the bubble to the liquid state occurs under the action of dynamic and thermophysical processes in the bubble–liquid system; the relative contribution of these processes is determined by the number Pe/Ja and is not the same at different phases of the process.

2. Relaxation of the collapsing vapor bubble is represenred by a number of sharp oscillations of the volume, which are of a shock nature, with subsequent increase in the amplitude of pressure and temperature at the end of each period of oscillations.

3. Overshoots in pressure in the phase of compression of the bubble are accompanied by a substantial superheating of the vapor, which causes intense heat withdrawal to the surrounding liquid. The phases of compression give way to short phases of expansion with supercooling of the vapor inside the bubble and repumping of heat from the liquid to the bubble.

4. The length of the phases of bubble expansion is less than that of the compression phases. Bubble expansion is accompanied by partial evaporation of the already condensed vapor.

5. The length of the compression phases is stipulated by external heat exchange of the bubble, which greatly decreases the rate of bubble collapse as compared with the rates of collapse in the purely inertia mode.

6. During the process of collapse, the bubble–liquid system passes a number of critical points on the curve flow rate–radius; each of these points produces the conditions for the hydraulic shock.

7. Deteriorated external heat exchange between the bubble and the surrounding liquid transfers the system to the mode of adiabatic damping oscillations and completely eliminates the possibility of hydraulic shock.

NOTATION

τ, time, sec; a, bubble radius, m; r, current radius, m; V, volume, m^3; l, specific heat of phase transition, J/kg; Q, amount of heat, J; λ, thermal conductivity, W/(m·K); T, temperature, K; ρ, substance density, kg/m^3; c, specific mass heat capacity, J/(kg·K); i, specific enthalpy, J/kg; p, absolute pressure, Pa; $\mathbf{q}$, vector of heat-flux density, W/m^2; $\mathbf{w}$, velocity vector, m/sec; γ, adiabatic index; v, specific volume, m^3/kg; χ, degree of dryness; ξ_0, droplet radius, m; R_μ, specific gas constant of vapor, J/(kg·K); μ, dynamic coefficient of liquid viscosity, Pa·sec; ν, kinematic coefficient of viscosity, m^2/sec; Σ, coefficient of surface tension, N/m; k, parameter of pressure disturbance; θ, reduced temperature in the liquid phase; ϑ, reduced temperature in the vapor phase; η, reduced space variable; M, N, grid parameters; L, infinitely remote point, m; A, volumetric flow rate of the phase interface, m^3/sec; α, heat-transfer coefficient, W/(m^2·K); E, kinetic energy, J; Π, elastic potential energy, J; E_{el}, volumetric elasticity modulus of the liquid phase, 1/Pa; Nu, Nusselt number; Ec, Eckert number; We, Weber number; Re, Reynolds number; Pe, Peclet number; Ja, Jacob numbers. Indices: 0, characteristic or initial value of the physical quantity; g, parameters of the gas or vapor phase; Σ, parameters on the phase interface; χ, constancy of the degree of dryness; a, kinematic parameters on the bubble surface; v, vapor, bubble; *, dimensionless parameters; ∞, parameters at infinity; s0, parameters of the dry saturated vapor; m, n, grid indices; cr, critical; el, elastic; max, maximum; primes, phase indices on the saturation curve; liq, liquid.

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